INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY RESEARCH AND ANALYSIS

ISSN(print): 2643-9840, ISSN(online): 2643-9875

Volume 05 Issue 12 December 2022

DOI: 10.47191/ijmra/v5-i12-34, Impact Factor: 6.261

Page No. 3539-3550

A Study on Network Graph-PW, Network Symmetric Digraph-PW, Change Network Graph-PW and Change Network Symmetric Digraph- PW

Abdu Alameri[1], Mohammed Alsharafi[2], Walid A.M. Saeed[3], Abdullatif Ghallab[4], Walid M. Yousef [5], Abdelhafid Modabish[6]

[1]Department of Biomedical Engineering, Faculty of Engineering, University of Science and Technology, Sana'a, Yemen

[2]Department of Mathematics, Faculty of Arts and Science, Yildiz Technical University, Istanbul, Turkey

[3]Department of Applied Mathematics, Faculty of Applied science, Taiz University, Taiz, Yemen

[4,5]Department of Information Technology, University of Science and Technology, Sana'a, Yemen

[6]Department of Mathematics, Faculty of Science, Sana'a University, Yemen

ABSTRACT: Networks play an important role in electrical and electronic engineering. It depends on what area of electrical and electronic engineering, for example, there is a lot more abstract mathematics in communication theory and signal processing and networking, etc. Networks involve nodes communicating with each other. Graph theory has found considerable use in this area. In this paper, we introduce some new Networks such as Graph-PW, Network Symmetric Digraph-PW, Change Network Graph-PW, and Change Network Symmetric Digraph- PW. Moreover, several theorems and results of these networks have been studied.

KEYWORDS: Graph-PW, Network Symmetric Digraph-PW, Change Network Graph-PW, Change Network Symmetric Digraph- PW.

INTRODUCTION

For more detail on Graphs, Digraphs, and network Digraphs one. The networks (Graph- *PW* and symmetric Digraph- *PW*) and network changes (Graph-*PW* and symmetric Digraph-*PW*) can be the source of many algorithms of practical importance. It is flexibly adaptable to suit the needs of the application, so it can be used in areas such as syntactic analysis, fault delectation, and diagnosis in computer, therefore the graphical of this representation of the object and the binary relation on them is a convenient form of expression. In this paper, we give some definitions and results of network Graphs-*PW* and symmetric Digraphs-*PW* and networks changes (Graph-*PW* and symmetric Digraph-*PW*) [1,2], we refer to the interested reader to [3-30].

1. Networks (Graph-PW And Symmetric Digraph-PW):

Definition (1.1): A **Graph-PW** is a triple $G_{PW} = (V, E^*, \Phi)$ consists of a non-empty set $V = \{v_1, v_2, \dots, v_n\}$ of objects called vertices, points, nodes, or just dots, together with undirected pairs set of vertices

$$E^* = \left\{ \begin{matrix} e_{ij} = v_i v_j = v_j v_i : i \neq j \\ or \ \ i = j, n(e_{ij}) = v_i * v_j = \rho\omega \end{matrix} \right\}$$

is called edges, arcs or lines, satisfy $PW = max(n(e_{ij}) = v_i * v_j = \rho\omega)$, where P (non-negative integer) is the maximum numbers of ρ-edges or loops between any pairs of vertices and W is the uniform weighted with $PW = max\rho\omega$ depended on ω uniform sign weighted in E^*, and incident function Φ from E^* to the set $P_2(V)$ of all 2-elements or parts subsets of V, that is, $\Phi: E^* \mapsto P_2(V)$. The **adjacency function matrix $Am(G_{PW})$** define as

$$Am(G_{PW}) = \begin{cases} n(e_{ij}) \ \ if \ \ \rho > 0 \wedge \omega > 0 \wedge v_i \ joined \ to \ v_j; \\ \ \ o \ or \ \infty \ \ if \qquad\qquad \rho\omega = 0 \qquad\qquad ; \\ -n(e_{ij}) \ \ if \ \rho > 0 \wedge \omega < 0 \wedge v_i \ joined \ to \ v_j. \end{cases}$$

clear that it is if $P = W = 1$, then the Graph-PW is natural **Graph-1**, and if $P > 0, W = 1$ the Graph-PW is **mullet Graph-P**, and if $P = 1, W \in \mathbb{R}^+$ the Graph-PW is **weighted Graph-W** and Φ in Graph-PW satisfy $\forall e_{ij} \in E^* \exists \{v_i, v_j\} \in P_2(V) \ni \Phi(e_{ij}) = \{v_i, v_j\}$, Φ is **one –to-one** if and only if $\Phi(e_{ij}) = \Phi(e_{rs}) \Longrightarrow e_{ij} = e_{rs}$ or $e_{ij} \neq e_{rs} \Longrightarrow \Phi(e_{ij}) \neq \Phi(e_{rs})$, **onto** if $\forall \{v_i, v_j\} \in$

$P_2(V) \exists e_{ij} \in E^* \ni \Phi(\{v_i, v_j\}) = e_{ij}$ or $\Phi(P_2(V)) = E^*$, and if Φ one-to-one and onto is called **corresponding** then there is an inverse Φ^{-1} of Φ.

The **operation** $*$ is modules, plus, product, max, min or … and the **examples** are only define the operation $*$ by these operations. In the following definitions and theorems PW, incident function and adjacency function matrix define as well as.

Definition (1.2): In the **network Graph-** $\boldsymbol{PW}$, the weight ω is called **flow function** for each edges of $\rho \in n(e_{ij})$ with $\omega_i \le \omega_{i+1}$ and $\rho = \sum_{i=1}^{\rho} \omega_i$ between any adjacent two pair vertices is called **capacity constraint** with $\omega \le \rho = \sum_{i=1}^{\rho} \omega_i$ and the total flow function for any vertex is $\omega(e) = \sum_{i=0} \omega_{i+1}$, and the total capacity constraint for any vertex is $\rho(e) = \sum_{j=1} \left(\sum_{i=1}^{\rho} \omega_i\right)_j$ with $\omega(e) \le \rho(e)$. If $\omega(e) = \sum_{i=0} \omega_{i+1} = PW$ then PW is called **value flow**, therefore, for any intermediate vertex if the total flow is PW of these vertices called **flow conservation**.

By these arguments the name **Graph-** $\boldsymbol{PW}$ and **network Graph-** $\boldsymbol{PW}$ are the same.

Definition (1.3): The **complement** of G_{PW} is $\bar{G}_{PW} = (V, \overline{E^*}, \bar{\Phi})$ with $V(G_{PW}) = V(\bar{G}_{PW})$, and $\overline{E^*} = \{e_{ij} = v_i v_j \wedge \bar{n}(e_{ij}) = PW - n(e_{ij}) \forall v_i, v_j \in V(G_{PW})\}$, and $\bar{\Phi}: \overline{E^*} \to P_2(V)$.
Therefore $G_{PW} \cup \bar{G}_{PW} = K_{|V|} - PW$ is called **complete Graph-** $\boldsymbol{PW}$**.**

Definition (1.4): The ***regular*** of G_{PW} is $R_{PW} = (V, E^*, \Phi)$ with
$E^* = \{e_{ij} = v_i v_j \colon deg\ v_i = deg v_j \wedge n(e_{ij}) = v_i * v_j = \rho\omega = PW\ \forall v_i, v_j \in V(G_{PW})\}$.
G_{PW} is $\textbf{semi}-\boldsymbol{regular}$ if one or two vertices have the equal degree different of all vertices, that is ,

$$(deg\ v_1 = deg v_2) \ne (deg\ v_i = deg v_j) \forall i, j \wedge n(e_{ij}) = v_i * v_j = PW$$

Theorem (1.1): If a Graph-PW $G_{PW} = (V, E^*, \Phi)$ or $\bar{G}_{PW}$ has an isolated vertex, then $G_{PW} - v$ or $G_{PW} \cup \bar{G}_{PW}$ has not an isolated vertex**.**

Proof: Since $G_{PW} = (V, E^*, \Phi)$ define on operations, then the operation module has only isolated vertex, and the operation of product has an isolated vertex, if one of vertex is zero, and the other operations $\bar{G}_{PW}$ has an isolated vertex, therefore $G_{PW} - v$ or $G_{PW} \cup \bar{G}_{PW}$ has not an isolated vertex.

Now, by these theorem $G_{PW}, \bar{G}_{PW}$ and $G_{PW} \cup \bar{G}_{PW}$ well define are connected and we can be taken these networks.

Definition (1.5): In the value flow of the networks, if $\sum_{i=0} \omega_{i+1} > PW$, then there is $\sum_{j=0} \omega_{j+1}$ such that $\sum_{i=0} \omega_{i+1} - \sum_{j=0} \omega_{j+1} = PW$, and $PW \le \rho(e) = \sum_{j=1} \left(\sum_{i=1}^{\rho} \omega_i\right)_j$**.**

Definition (1.6): A **sub-Graphs-**$\dot{\boldsymbol{P}}\dot{\boldsymbol{W}}$ of a Graph-PW is H_{pw} or $\dot{G}_{PW} = (V, E^*, \Phi) \subseteq G_{PW}$ with $V(H_{pw}) \subseteq V(G_{PW})$ or $V(\dot{G}_{PW}) = V(G_{PW}), E^*(H_{pw})$ or
$E^*(\dot{G}_{PW}) \subseteq E^*(G_{PW})$, and $\Phi(H_{pw})$ or $\Phi(\dot{G}_{PW}) \subseteq \Phi(G_{PW})$.
Now, let $H_{pw} = (V(H), E^*(H), \Phi(H)), Y_{pw} = (V(Y), E^*(Y), \Phi(Y)) \subseteq G_{PW} = (V, E^*, \Phi)$ with $V(Y) = V(G_{PW}) - V(H), E^*(Y) = \{e_{ij} = v_i v_j \colon n(e_{ij}) = v_i * v_j = \rho\omega \notin E^*(H)\}$. Then $H_{pw} \cup Y_{pw} \subseteq G_{PW}$, and $H_{pw} + Y_{pw} = G_{PW}$, where $V(H) \cup V(Y) = V(G_{PW})$ and $E^*(H) \cup E^*(Y) \cup \{e_{ij} = v_i v_j \colon v_i \in V(H), v_j \in V(Y) \forall v_i, v_j, n(e_{ij}) = v_i * v_j = \rho\omega\}$.

Let $H_{pw} = (V(H), E^*(H), \Phi(H)), Y_{pw} = (V(Y), E^*(Y), \Phi(Y)) \subseteq \dot{G}_{PW} = (V, E^*, \Phi) \subseteq G_{PW}$ with $V(Y) = V(\dot{G}) - V(H), E^*(Y) = \{e_{ij} = v_i v_j \colon n(e_{ij}) = v_i * v_j = \rho\omega \notin E^*(H)\}$. Then $H_{pw} \cup Y_{pw} \subseteq \dot{G}_{PW}$, and

$$H_{pw} \cup Y_{pw} = \dot{G}_{PW}\ iff$$

$E^*(H) \cup E^*(Y) \cup \{e_{ij} = v_i v_j \colon v_i \in V(H), v_j \in V(Y) \wedge n(e_{ij}) = v_i * v_j = \rho\omega \in E^*(\dot{G}_{PW})\}$.

Now, we can give the following results on the network Graph-PW on flow function, value flow and capacity constraint.

Theorem (1.2): If $H_{pw} = \left(V(H_{pw}), E^*(H_{pw}), \Phi(H_{pw})\right) \subseteq G_{PW} = (V, E^*, \Phi)$ of network Graph-PW, with $pw = PW$, then $H_{pw} + G_{PW} - V(H_{pw}) = G_{PW}$.

Proof: By definition (1.6) the result follows.

Theorem (1.3): Let $\omega(e) = \sum_{i=0} \omega_{i+1}$, $\rho(e) = \sum_{j=1}(\sum_{i=1}^{\rho} \omega_i)_j > PW$ be total flow function and capacity constraint of networks Graph- PW. Then there is
$\sum_{r=0} \omega_{i+1}(e), \rho(e) = \sum_{r=1}(\sum_{i=1}^{\rho} \omega_i)_r \in H_{pw}$ or $G_{PW} - V(H_{pw}) \subseteq G_{PW} = (V, E^*, \Phi)$ such that

I. $\omega(e) - \sum_{r=0} \omega_{r+1} = PW$, and

II. $\rho(e) = \sum_{j=1} \left(\sum_{i=1}^{\rho} \omega_i\right)_j - \rho(e) = \sum_{r=1} \left(\sum_{i=1}^{\rho} \omega_i\right)_r = PW$.

Proof: I. Since $\sum_{r=0} \omega_{i+1} \in H_{pw}$ or $G_{PW} - V(H_{pw})$, then $\sum_{r=0} \omega_{r+1} \in G_{PW}$, and by definition (1.5) the result follows, and

II. Since $\rho(e) = \sum_{r=1} \left(\sum_{i=1}^{\rho} \omega_i\right)_r \in H_{pw}$ or $G_{PW} - V(H_{pw})$, then
$\rho(e) = \sum_{r=1} \left(\sum_{i=1}^{\rho} \omega_i\right)_r \in G_{PW}$, and the result follows.

Theorem (1.4): Let $\omega(e) = \sum_{i=0} \omega_{i+1}$, $\rho(e) = \sum_{j=1}(\sum_{i=1}^{\rho} \omega_i)_j$ be total flow function and capacity constraint of networks Graph- PW have value flow and let, $\omega_j(e), \rho_j(e)$ be total flow function and capacity constraint of sub-graph- pw from H_{pw} to $G_{PW} - V(H_{pw})$. Then $PW = \omega_j(e) - \omega_r(e)$, $PW = \rho_j(e)$- $\rho_r(e)$, where $\omega_r(e)$ $\rho_r(e)$ are the total flow function and capacity constraint from $G_{PW} - V(H_{pw})$ to H_{pw}. Moreover, $W \leq \rho_j(e)$.

Proof: I. Since $\omega(e) = \sum_{i=0} \omega_{i+1}, \rho(e) = \sum_{j=1}(\sum_{i=1}^{\rho} \omega_i)_j \in G_{PW}$, then the total flow function and capacity constraint from H_{pw} or $G_{PW} - V(H_{pw})$ to G_{PW} equal the total flow function and capacity constraint from G_{PW} to H_{pw} or $G_{PW} - V(H_{pw})$, and
$H_{pw} + G_{PW} - V(H_{pw}) = G_{PW}$, then $PW = \omega_j(e) - \omega_r(e)$, $PW = \rho_j(e)$- $\rho_r(e)$
Moreover, $\omega_j(e) - \omega_r(e) \leq \omega_j(e) \leq \rho_j(e)$.

Definition (1.7): Let $\dot{G}_{PW} = (V, E^*(\dot{G}_{PW}), \Phi), \ddot{G}_{PW} = (V, E^*(\ddot{G}_{PW}), \Phi(\ddot{G}_{PW}) \subseteq G_{PW}$ be sub $-$ Graph $- \dot{P}\dot{W}$ and sub $-$ Graph $- \ddot{P}\ddot{W}$ of network Graph $- PW$ with
$V = \{v_i : i = 1,2,3, \dots, n\}$, and
$E^*(\dot{R}_{PW}) = \{e_{ij} = v_i v_j : deg\, v_i = deg v_j \ \forall i,j \ \wedge \ n(e_{ij}) = v_i * v_j = \dot{P}\dot{W}\}$, and
$E^*(\ddot{R}_{PW}) = \{e_{ij} = v_i v_j : deg\, v_i = deg v_j \ \forall i,j \ \wedge \ n(e_{ij}) = v_i * v_j = \ddot{P}\ddot{W} \notin E^*(\dot{G}_{PW})\}$.
Then $\dot{G}_{PW}$ and $\ddot{G}_{PW}$ are called **Regulars sub-graph-$\dot{P}\dot{W}$** and **sub $-$ Graph $- \ddot{P}\ddot{W}$** denoted by $\dot{R}_{PW}$ and $\ddot{R}_{PW}$**.** $\dot{G}_{PW}$ and $\ddot{G}_{PW}$ are **semi $-$ reglars** if one or two vertices have the equal degree different of all vertices, that is, $(deg\, v_1 = deg v_2) \neq (\,deg\, v_i = deg v_j) \forall i,j \ \wedge \ n(e_{ij}) = v_i * v_j = \dot{P}\dot{W} \vee \ddot{P}\ddot{W}$.

Theorem (1.5): $\dot{R}_{PW} = (V, E^*, \Phi)$ is regular sub $-$ graph $- \dot{P}\dot{W}\, iff\ \ddot{R}_{PW}\ or\ \dot{R}_{PW} \cup \ddot{R}_{PW}$ are regulars.
Moreover, if $\dot{R}_{PW}$ is semi $-$ reglar sub $-$ Graph $- \dot{P}\dot{W}$, then $\ddot{R}_{PW} \vee \dot{R}_{PW} \cup \ddot{R}_{PW}$ are semi $-$ reglars or regulars.
Therefore, $\dot{R}_{PW} = \ddot{R}_{PW}\ iff\ \dot{P}\dot{W} = \ddot{P}\ddot{W}$.

Proof: See definition (1,7).

Definition (1.8): Let $\dot{G}_{PW}, \ddot{G}_{PW} \subseteq G_{PW}$ be Graph $- \dot{P}\dot{W}$ and Graph $- \ddot{P}\ddot{W}$ of network Graph $- PW$ with $V(\dot{G}_{PW}) = V(\ddot{G}_{PW}) = V(G_{PW}), E(\dot{G}_{PW}) \cap E(\ddot{G}_{PW}) = \varphi$, and
$\dot{G}_{PW} \cup \ddot{G}_{PW} = G_{PW}$. Then $\bar{\dot{G}}_{PW} = \left(V(G_{PW}), E^*\left(\bar{\dot{G}}_{PW}\right), \bar{\dot{\Phi}}\right)$ where

$$E^*\left(\bar{\dot{G}}_{PW}\right) = E^*(\ddot{G}_{PW}) \cup \{\bar{n}(e_{ij}) : \bar{n}(e_{ij}) = \dot{P}\dot{W} - n(e_{ij}), n(e_{ij}) \in E^*(\dot{G}_{PW})\},\text{ and}$$

$$\bar{\ddot{G}}_{PW} = \left(V(G_{PW}), E^*\left(\bar{\ddot{G}}_{PW}\right), \bar{\ddot{\Phi}}\right)\text{ where}$$

$E^*\left(\bar{\ddot{G}}_{PW}\right) = E^*(\dot{G}_{PW}) \cup \{\bar{n}(e_{ij}) : \bar{n}(e_{ij}) = \ddot{P}\ddot{W} - n(e_{ij}), n(e_{ij}) \in E^*(\ddot{G}_{PW})\}$.

Theorem (1.6): $\bar{\dot{G}}_{PW} \cup \bar{\ddot{G}}_{PW}$=($V(G_{PW}), E^*\left(\bar{\dot{G}}_{PW}\right) \cup E^*\left(\bar{\ddot{G}}_{PW}\right), \bar{\dot{\Phi}} \cup \bar{\ddot{\Phi}}$).
Proof: By definition (1.8) the result follows.

Theorem (1.7): 1. $\overline{\dot{G}_{PW} \cup \ddot{G}_{PW}} = \bar{G}_{PW} = \bar{\dot{G}}_{PW} \cap \bar{\ddot{G}}_{PW}$, with $\dot{G}_{PW} \cup \ddot{G}_{PW} = G_{PW}$

2. $\bar{\dot{G}}_{PW} \cup \bar{\ddot{G}}_{PW} = G_{PW} = \overline{\dot{G}_{PW} \cap \ddot{G}_{PW}}$,with

$E^*\left(\bar{\dot{G}}_{PW} \cup \bar{\ddot{G}}_{PW}\right) = \{\max\left(n(e_{ij}), \bar{n}(e_{ij})\right) : n(e_{ij}) \in E^*(\dot{G}_{PW}) or\ E^*(\ddot{G}_{PW})$and $\bar{n}(e_{ij}) \in E^*\left(\bar{\dot{G}}_{PW}\right) or E^*\left(\bar{\ddot{G}}_{PW}\right)\}$.

Proof: 1. Since $\dot{G}_{PW} \cup \ddot{G}_{PW} = G_{PW}$, then $\overline{\dot{G}_{PW} \cup \ddot{G}_{PW}} = \bar{G}_{PW}$, and by Morgan laws $\overline{\dot{G}_{PW} \cup \ddot{G}_{PW}} = \bar{\dot{G}}_{PW} \cap \bar{\ddot{G}}_{PW}$, and

2. $\bar{\dot{G}}_{PW} \cup \bar{\ddot{G}}_{PW} = G_{PW} = \overline{\dot{G}_{PW} \cap \ddot{G}_{PW}}$ (by Morgan laws).

Definition (1.9): Let $\dot{H}_{PW}, \ddot{H}_{PW} \subseteq G_{PW}$ be Graph $- \dot{P}\dot{W}$ and Graph $- \ddot{P}\ddot{W}$ network Graph $- PW$with $V(\dot{H}_{PW}) = V(\ddot{H}_{PW}), E(\dot{H}_{PW}) \cap E(\ddot{H}_{PW}) = \varphi$, and $\dot{H}_{PW} \cup \ddot{H}_{PW} = G_{PW}$.Then $\dot{H}_{PW} = (V(G_{PW}), E^*(\dot{H}_{PW}), \Phi(\dot{H}_{PW}))$ where

$$E^*(\dot{H}_{PW}) = E^*(\ddot{H}_{PW}) \cup \{\bar{n}(e_{ij}): \bar{n}(e_{ij}) = max(\dot{P}\dot{W}, \ddot{P}\ddot{W}) - n(e_{ij}), n(e_{ij}) \in E^*(\dot{H}_{PW})\}$$

and $\ddot{H}_{PW} = (V(G_{PW}), E^*(\ddot{H}_{PW}), \Phi(\ddot{H}_{PW}))$where

$E^*(\ddot{H}_{PW}) = E^*(\dot{H}_{PW}) \cup \{\bar{n}(e_{ij}): \bar{n}(e_{ij}) = \max(\dot{P}\dot{W}, \ddot{P}\ddot{W}) - n(e_{ij}), n(e_{ij}) \in E^*(\ddot{H}_{PW})\}$

Theorem (1.8): $\dot{H}_{PW} \cup \ddot{H}_{PW} = G_{PW} \cup \bar{G}_{PW}$.

Proof: By definition (1.3) and (1.9), the result follows.

Definition (1.10): Let $\dot{Y}_{PW}, \ddot{Y}_{PW} \subseteq G_{PW}$ be Graph $- \dot{P}\dot{W}$ and Graph $- \ddot{P}\ddot{W}$ of network Graph $- PW$with $\dot{Y}_{PW} = (V(G_{PW}), E^*(\dot{Y}_{PW}), \Phi(\dot{Y}_{PW}))$,

$E^*(\dot{Y}_{PW}) = \{\bar{n}(e_{ij}): \bar{n}(e_{ij}) = \dot{P}\dot{W} - n(e_{ij})\ \forall v_i, v_j \in V(G_{PW})\}$ and

$\ddot{Y}_{PW} = (V(G_{PW}), E^*(\ddot{Y}_{PW}), \Phi(\ddot{Y}_{PW}))$,

$E^*(\ddot{Y}_{PW}) = \{\bar{n}(e_{ij}): \bar{n}(e_{ij}) = \ddot{P}\ddot{W} - n(e_{ij})\ \forall v_i, v_j \in V(G_{PW})\}$.

Theorem (1.9): $\dot{G}_{PW} \cup \dot{Y}_{PW}$and $\ddot{G}_{PW} \cup \ddot{Y}_{PW}$are complete Graph $- \dot{P}\dot{W}$ and Graph-$\ddot{P}\ddot{W}$.

Proof: By definition (1.3) and (1.10)$G_{PW} \cup \bar{G}_{PW} = \dot{G}_{PW} \cup \dot{Y}_{PW}$ or $\ddot{G}_{PW} \cup \ddot{Y}_{PW}$with $PW = \dot{P}\dot{W}$and $PW < \ddot{P}\ddot{W}$ or $PW = \ddot{P}\ddot{W}$and $PW < \dot{P}\dot{W}$which are completes.

Definition (1.11): Let $\dot{G}_{PW} = (V, E^*, \Phi) \subseteq G_{PW}$ be connected Graph $- \dot{P}\dot{W}$ with loops and $V(\dot{G}_{PW}) = V(G_{PW})$ and let $\dot{G}^2_{PW} = (V, E^*_2, \Phi_2)$ define as-well- as $\dot{G}_{PW}$ with joint two vertices v_i, v_jnon-adjacency by $n(e_{ij}) = v_i * v_j$ with $n(e_{ir})$ and $n(e_{rj})$ adjacency by v_r, that is, equivalence $1 \leq d(v_i, v_j) \leq 2$, where v_i is adjacent to $v_j$in $\dot{G}_{PW}$, $d(v_i, v_j)$ is the minimum numbers of vertices except first or last vertex between v_i, v_j. We can continuous to define $\dot{G}^3_{PW} = (V, E^*_3, \Phi_3)$, … , then, $\dot{G}^r_{PW} = (V, E^*_r, \Phi_r)$ is called a **Power Graph-$\dot{P}\dot{W}$**.

Theorem (1.10): $\dot{G}^r_{PW} = G_{PW} iff\ r = maxd(v_i, v_j)$

Proof: Since $\dot{G}_{PW} = (V, E^*, \Phi) \subseteq G_{PW}$with the loops, then$\dot{G}^2_{PW} \subseteq G_{PW}$, and so on $\dot{G}^{r-1}_{PW} \subseteq G_{PW}$, and $\dot{G}^r_{PW} = G_{PW} iff\ r = maxd(v_i, v_j)$.

Now, $\dot{R}_{pW}$, $\bar{\dot{G}}_{PW}$, $\bar{\ddot{G}}_{PW} \dot{H}_{PW}$, $\ddot{H}_{PW}$, $\dot{Y}_{PW}$, $\ddot{Y}_{PW}$, and$\dot{G}^r_{PW}$well define connected network Graph-$\dot{P}\dot{W}$and Graph-$\ddot{P}\ddot{W}$.

Definition (1.12) A **network symmetric Directed Graph-*PW*** is triple $SD_{PW} = (V, A^*, \Phi)$ consists of a non-empty Set $V = \{v_1, v_2, \dots, v_n\}$ of objects called vertices, points, nodes, or just dots, together with directed pairs set of vertices

$$A^* = \begin{Bmatrix} \mathrm{a} = (a_{ij} = (v_i, v_j) \cup (v_j, v_i) = a_{ji})) : i \neq j\ or\ i = j\ \wedge \\ n(\mathrm{a}_{ij}) = v_i * v_j = v_j * v_i = \rho\omega \end{Bmatrix}$$

is called edges, arcs or lines, satisfy $PW = \max(n(\mathrm{a}) = v_i * v_j = \rho\omega)$, where $\boldsymbol{P}$ (non-negative integer) is the maximum numbers of ρ-arcs or loops between any pairs of vertices and W is the uniform weighted with $PW = max\rho\omega$ depended on ω uniform sign weighted in $\omega\ in\ A^*$, and incident function Φ from A to the set V^2,that is, $\Phi: A^* \mapsto V^2$. The **adjacency function matrix $\boldsymbol{Am}(\boldsymbol{SD_{PW}})$** define as

$$Am(SD_{PW}) = \begin{cases} n(\mathrm{a}) \ \ if \ \ P > 0 \wedge w > 0 \wedge v_i \ joined \ to \ v_j; \\ o \ or \ \infty \ \ if \qquad\qquad \rho\omega = 0 \ \ ; \\ -n(\mathrm{a}) \ \ if \ \ P > 0 \wedge w < 0 \wedge v_i \ joined \ to \ v_j. \end{cases}$$

clear that it is if $P = W = 1$, then the symmetric Digraph- PW is natural **symmetric Digrahp-1**, and if $P > 0, W = 1$ the symmetric Digraph- PW is **mullet symmetric Digraph- P**, and if $P = 1, W \in \mathbb{R}^+$ the symmetric Digraph- PW is **weighted symmetric Digraph- W** and Φ in symmetric Digraph- PW satisfy
$\forall \mathrm{a} \in A^* \exists \{(v_i, v_j), (v_j, v_i)\} \in P_2(V) \ni \Phi(\mathrm{a}) = \{(v_i, v_j), (v_j, v_i)\}$, $\Phi$is one –to-one if and only if $\Phi(\mathrm{a}_i) = \Phi(\mathrm{a_j}) \Longrightarrow \mathrm{a}_i = \mathrm{a_j}$ or $\mathrm{a}_i \neq \mathrm{a_j} \Longrightarrow \Phi(\mathrm{a}_i) \neq \Phi(\mathrm{a_j})$, onto if $\forall \{(v_i, v_j), (v_j, v_i)\} \in P_2(V) \exists \mathrm{a} \in A^* \ni \Phi(\{(v_i, v_j), (v_j, v_i)\}) = \mathrm{a} \ or \ \Phi(P_2(V)) = A^*$, and if Φ one-to-one and onto is called corresponding then there is an inverse Φ^{-1} of Φ.

In the following definitions and theorems, incident function and adjacency function matrix define as well as.

Definition (1.13): In the **network symmetric Digraph- PW**, the weight ω is called **symmetric flow function** for each arcs of $\rho \in n(\mathrm{a})$ with $\omega_i \le \omega_{i+1}$ and $\rho = \sum_{i=1}^{\rho} \omega_i$ between any adjacent two pair vertices is called **symmetric capacity constraint** with $\omega \le \rho = \sum_{i=1}^{\rho} \omega_i$ a and the total symmetric flow function for any vertex is
$\omega(\mathrm{a}) = \sum_{i=0} \omega_{i+1}$, and the total symmetric capacity constraint for any vertex is $\rho(\mathrm{a}) = \sum_{j=1} (\sum_{i=1}^{\rho} \omega_i)_j$ with $\omega(\mathrm{a}) \le \rho(\mathrm{a})$. If $\omega(\mathrm{a}) = \sum_{i=0} \omega_{i+1} = PW$ then PW is called **value symmetric flow**, therefore, for any intermediate vertex if the total symmetric flow is PW of these vertices is called **symmetric flow conservation**.

By these arguments the name **symmetric Digraph- PW**and **network symmetric Digraph- PW**are the same.

Definition (1.14): The **complement** of SD_{PW} is $S\overline{D}_{PW} = (V, \overline{A^*}, \overline{\Phi})$ with
$V(SD_{PW}) = V(S\overline{D}_{PW})$, and
$\overline{A^*} = \{\mathrm{a} = (v_i \ v_\mathrm{j}) \cup (v_j \ v_i) \ \wedge \ \bar{n}(\mathrm{a}) = PW - \ n(\mathrm{a}) \forall v_i, v_j \in V(SD_{PW})$, and
$\overline{\Phi}: \overline{A^*} \to V^2$. Therefore $SD_{PW} \cup \ S\overline{D}_{PW} = \ K_{|V|} - PW$ is called **complete symmetric Digraph- PW.**

Definition (1.15): The **regular** of $SD_{PW}$is $SR_{PW} = (V, A^*, \Phi)$ with
$A^* = \{\mathrm{a} = (v_i \ v_\mathrm{j}) \cup (v_j v_i)\text{: deg } v_i = deg v_j \wedge n(\mathrm{a}) = v_i * v_\mathrm{j} = \rho\omega = PW \ \forall v_i, v_j \in V(SD_{PW})\}$
and SD_{PW} is $\mathbf{sime-regular}$ if
$(indeg \ v_1 = indeg v_2) \neq (\ indeg \ v_i = indeg v_j) \ \forall i, j \ \wedge \ \ n(\mathrm{a}) = v_i * v_j = PW$

Theorem (1.12): If a symmetric Digraph- PW $SD_{PW} = (V, A^*, \Phi)$ or $S\overline{D}_{PW}$has an isolated vertex, then $SD_{PW} - v$ or $SD_{PW} \cup \ S\overline{D}_{PW}$ has not a isolated vertex**.**

Proof: Since $SD_{PW} = (V, A^*, \Phi)$ is defined on operations, then the operation module has only isolated vertex, and the operation of product has isolated vertex, if one of vertex is zero, and the anther operations $S\overline{D}_{PW}$ has isolated vertex, therefore$SD_{PW} - v$
or
$SD_{PW} \cup \ S\overline{D}_{PW}$ has not an isolated vertex.

Now, by these theorem SD_{PW},$S\overline{D}_{PW}$ and $SD_{PW} \cup \ \bar{G}_{PW}$ well define are connected and we can be taken these network.

Definition (1.16): In the value flow of the networks, if $\sum_{i=0} \omega_{i+1} \ > PW$, then there is $\sum_{j=0} \omega_{j+1}$ such that $\sum_{i=0} \omega_{i+1} - \sum_{j=0} \omega_{j+1} = PW$, and$PW \le \rho(\mathrm{a}) = \sum_{j=1} (\sum_{i=1}^{\rho} \omega_i)_j$.

Definition (1.17): A **sub-symmetric Digraphs-$\dot{P}\dot{W}$** of a symmetric Digraph-PW is SH_{pw} or $S\dot{D}_{PW} = (V, A^*, \Phi) \subseteq SD_{PW}$ with$V(SH_{pw}) \subseteq V(SD_{PW})$ or
$V(S\dot{D}_{PW}) = V(SD_{PW})$, $A^*(SH_{pw})$ or
$A^*(S\dot{D}_{PW}) \subseteq A^*(SD_{PW})$,and$\Phi(SH_{pw})$ or $\Phi(\ S\dot{D}_{PW}) \subseteq \Phi(SD_{PW})$.
Now, let $SH_{pw} = (V(SH), A^*(SH), \Phi(SH))$and
$SY_{pw} = (V(SY), A^*(SY), \Phi(SY)) \ \subseteq SD_{PW} = (V, A^*, \Phi)$ with

$$V(SY) = V(SD_{PW}) - V(SH),$$

$A^*(Y) = \{\mathrm{a} = (v_i, v_j) \cup (v_j\,, v_i): n(\mathrm{a}) = v_i * v_j = \rho\omega \notin A^*(SH)\}$. Then
$SH_{pw} \cup SY_{pw} \subseteq SD_{PW}$, and $SH_{pw} + SY_{pw} = SD_{PW}$, where $V(SH) \cup V(SY) = V(SD_{PW})$and $A^*(SH) \cup A^*(SY) \cup \{\mathrm{a} = (v_i, v_j) \cup (v_j, v_i)$: $v_i \in V(H), v_j \in V(Y) \forall v_i, v_j$,
$n(\mathrm{a}) = v_i * v_j = \rho\omega\}$.
Let $SH_{pw} = (V(SH), A^*(SH), \Phi(SH))$,
$SY_{pw} = (V(SY), A^*(SY), \Phi(SY)) \subseteq S\dot{D}_{PW} = (V, A^*, \Phi) \subseteq SD_{PW}$ with

$$V(SY) = V(S\dot{D}) - V(SH),$$

$A^*(Y) = \{\mathrm{a} = (v_i, v_j) \cup (v_j, v_i): n(\mathrm{a}) = v_i * v_j = \rho\omega \notin A^*(SH)\}$. Then
$SH_{pw} \cup SY_{pw} \subseteq S\dot{D}_{PW}$, and $SH_{pw} \cup SY_{pw} = S\dot{D}_{PW} iff$

$$A^*(SH) \cup A^*(SY) \cup \{\mathrm{a} = (v_i, v_j) \cup (v_j\,, v_i): v_i \in V(SH), v_j \in V(SY) \wedge$$

$n(\mathrm{a}) = v_i * v_j = \rho\omega \in A^*(S\dot{D}_{PW})\}$.

Now, we can give the following results on the network symmetric Digraph-PW on symmetric flow function, value symmetric flow and symmetric capacity constraint.

Theorem (1.13): If $SH_{pw} = \left(V(SH_{pw}), A^*(SH_{pw}), \Phi(SH_{pw})\right) \subseteq SD_{PW} = (V, A^*, \Phi)$ of network symmetric Digraph-PW, with $pw = PW$, then
$SH_{pw} + SD_{PW} - V(SH_{pw}) = SD_{PW}$.

Proof: By definition (1.17) the result follows.

Theorem (1.14): Let $\omega(\mathrm{a}) = \sum_{i=0} \omega_{i+1}$, $\rho(\mathrm{a}) = \sum_{j=1}(\sum_{i=1}^{\rho} \omega_i)_j > PW$ be total symmetric flow function and symmetric capacity constraint of networks symmetric Digraph-PW. Then there is
$\sum_{r=0} \omega_{i+1}, \rho(\mathrm{a}) = \sum_{r=1}(\sum_{i=1}^{\rho} \omega_i)_r \in SH_{pw}$or $SD_{PW} - V(SH_{pw}) \subseteq SD_{PW} = (V, A^*, \Phi)$ such that

I. $\omega(\mathrm{a}) - \sum_{r=0} \omega_{r+1} = PW$, and

II. $\rho(\mathrm{a}) = \sum_{j=1} \left(\sum_{i=1}^{\rho} \omega_i\right)_j - \rho(\mathrm{a}) = \sum_{r=1} \left(\sum_{i=1}^{\rho} \omega_i\right)_r = PW$.

Proof: I. Since $\sum_{r=0} \omega_{i+1} \in SH_{pw}$or $SD_{PW} - V(SH_{pw})$, then $\sum_{r=0} \omega_{r+1} \in SD_{PW}$, and by definition (1.5) the result follows, and

II. Since $\rho(\mathrm{a}) = \sum_{r=1} \left(\sum_{i=1}^{\rho} \omega_i\right)_r \in SH_{pw}$or $SD_{PW} - V(SH_{pw})$,then
$\rho(\mathrm{a}) = \sum_{r=1} \left(\sum_{i=1}^{\rho} \omega_i\right)_r \in SD_{PW}$, and the result follows.

Theorem (1.15): Let $\omega(\mathrm{a}) = \sum_{i=0} \omega_{i+1}$, $\rho(\mathrm{a}) = \sum_{j=1}(\sum_{i=1}^{\rho} \omega_i)_j$ be total symmetric flow function and symmetric capacity constraint of networks symmetric Digraph-PW have value symmetric flow and let, $\omega_j(\mathrm{a}), \rho_j(\mathrm{a})$ be total symmetric flow function and symmetric capacity constraint of sub- symmetric Digraph- pw from SH_{pw} to $SD_{PW} - V(SH_{pw})$. Then$PW = \omega_j(\mathrm{a}) - \omega_r(\mathrm{a}), PW = \rho_j(\mathrm{a})$- $\rho_r(\mathrm{a})$, where $\omega_r(\mathrm{a})\ \rho_r(\mathrm{a})$ are the total symmetric flow function and symmetric capacity constraint from $G_{PW} - V(H_{pw})$ toH_{pw}.Moreover, $\leq \rho_j(\mathrm{a})$.

Proof: Since $\omega(\mathrm{a}) = \sum_{i=0} \omega_{i+1}, \rho(\mathrm{a}) = \sum_{j=1}(\sum_{i=1}^{\rho} \omega_i)_j \in G_{PW}$, then the total symmetric flow function and symmetric capacity constraint from SH_{pw} or $SD_{PW} - V(SH_{pw})$to SD_{PW} equal the total symmetric flow function and symmetric capacity constraint from SD_{PW} to SH_{pw} or $SD_{PW} - V(SH_{pw})$, and
$SH_{pw} + SD_{PW} - V(SH_{pw}) = SD_{PW}$, then $PW = \omega_j(\mathrm{a}) - \omega_r(\mathrm{a}), PW = \rho_j(\mathrm{a})$- $\rho_r(\mathrm{a})$
Moreover, $\omega_j(\mathrm{a}) - \omega_r(\mathrm{a}) \leq \omega_j(\mathrm{a}) \leq \rho_j(\mathrm{a})$.

Definition (1.18): Let $S\dot{D}_{PW} = (V, A^*(S\dot{D}_{PW}), \Phi)$and

$$S\ddot{D}_{PW} = (V, A^*(S\ddot{D}_{PW}), \Phi(S\ddot{D}_{PW})) \subseteq SD_{PW}$$

$$\text{be sub} - \text{symmetricDigraph} - \dot{P}\dot{W}\text{and sub} - \text{symmetric Digraph} - \ddot{P}\ddot{W}\text{of network}$$

$\text{symmetric Digraph} - PW$ with $V = \{v_i: i = 1,2,3, \dots, n\}$, and

$A^*(S\dot{R}_{PW}) = \{a = v_i v_j:\ deg\ v_i = deg v_j\ \forall i,j\ \wedge\ n(a) = v_i * v_j = \dot{P}\dot{W}\}$, and
$A^*(S\ddot{R}_{PW}) = \{a = v_i v_j:\ indeg\ v_i = outdeg v_j\ \forall i,j\ \wedge$
$n(a) = v_i * v_j = \ddot{P}\ddot{W} \notin E^*(S\dot{D}_{PW})\}$.

Then $S\dot{D}_{PW}$ and $S\ddot{D}_{PW}$ are called **Regulars sub- symmetric Digraph-$\dot{P}\dot{W}$** and $\mathbf{sub-symmetric\ Digraph}-\ddot{P}\ddot{W}$ denoted by $S\dot{R}_{PW}$ and $S\ddot{R}_{PW}$. $S\dot{D}_{PW}$ and $S\ddot{D}_{PW}$ are $\mathbf{semi-regular}$ if one or two vertices have the equal degree different of all vertices, that is ,
$(indeg\ v_1 = indeg v_2) \neq (\ indeg\ v_i = indeg v_j)\forall i,j\ \wedge\ n(a) = v_i * v_j = \dot{P}\dot{W} \vee \ddot{P}\ddot{W}$.

Theorem (1.15): $\dot{R}_{PW} = (V, A^*, \Phi)$ is regular sub $-$ symmetric Digraph $- \dot{P}\dot{W}\, iff\ S\ddot{R}_{PW}$ $or\ S\dot{R}_{PW} \cup S\ddot{R}_{PW}$ are regulars. Moreover, if $S\dot{R}_{PW}$ is semi $-$ reglar sub $-$ symmetric Digraph $- \dot{P}\dot{W}$, then $S\ddot{R}_{PW}\ or S\dot{R}_{PW} \cup S\ddot{R}_{PW}$ are semi $-$ reglars or regulars. Therefore, $S\dot{R}_{PW} = S\ddot{R}_{PW}\ iff\ \dot{P}\dot{W} = \ddot{P}\ddot{W}$.

Proof: See definition (1,7).

Definition (1.19): Let $S\dot{D}_{PW}, S\ddot{D}_{PW} \subseteq SD_{PW}$ be Graph $- \dot{P}\dot{W}$ and Graph $- \ddot{P}\ddot{W}$ of network Graph $- PW$ with $V(S\dot{D}_{PW}) = V(S\ddot{D}_{PW}) = V(SD_{PW}), A^*(\ S\dot{D}_{PW}) \cap A^*(S\ddot{D}_{PW}) = \varphi$, and $S\dot{D}_{PW} \cup S\ddot{D}_{PW} = SD_{PW}$.Then $S\bar{\dot{D}}_{PW} = \left(V(SD_{PW}), A^*\left(S\bar{\dot{D}}_{PW}\right), \bar{\dot{\Phi}}\right)$ where

$$A^*\left(S\bar{\dot{D}}_{PW}\right) = A^*(S\ddot{D}_{PW}) \cup \{\bar{n}(a): \bar{n}(a) = \dot{P}\dot{W} - n(a), n(a)\ \in A^*(\ S\dot{D}_{PW})\},\text{ and}$$
$$S\bar{\ddot{D}}_{PW} = \left(V(SD_{PW}), A^*\left(\bar{\ddot{G}}_{PW}\right), \bar{\ddot{\Phi}}\right)\text{ where}$$
$$A^*\left(S\bar{\ddot{D}}_{PW}\right) = A^*(\ S\dot{D}_{PW}) \cup \{\bar{n}(a): \bar{n}(a) = \ddot{P}\ddot{W} - n(a), n(a)\ \in A^*(\ S\ddot{D}_{PW})\}.$$

Theorem (1.16): $S\bar{\dot{D}}_{PW} \cup S\bar{\ddot{D}}_{PW}$=($V(SD_{PW}), A^*\left(S\bar{\dot{D}}_{PW}\right) \cup A^*\left(S\bar{\ddot{D}}_{PW}\right), \bar{\dot{\Phi}} \cup \bar{\ddot{\Phi}}$).

Proof: By definition (1.8) the result follows.

Theorem (1.17): 1. $\overline{S\dot{D}_{PW} \cup S\ddot{D}_{PW}} = S\bar{D}_{PW} = S\bar{\dot{D}}_{PW} \cap S\bar{\ddot{D}}_{PW}$, with $S\dot{D}_{PW} \cup S\ddot{D}_{PW} = SD_{PW}$

2. $S\bar{\dot{D}}_{PW} \cup S\bar{\ddot{D}}_{PW} = SD_{PW} = \overline{S\dot{D}_{PW} \cap S\ddot{D}_{PW}}$,with

$$A^*\left(\ S\bar{\dot{D}}_{PW} \cup S\bar{\ddot{D}}_{PW}\right) = \{\max(n(a), \bar{n}(a))\ :\ n(a) \in A^*(\ S\dot{D}_{PW}) or\ A^*(\ S\ddot{D}_{PW})\text{and}$$
$$\bar{n}(a) \in A^*\left(S\bar{\dot{D}}_{PW}\right) or A^*\left(S\bar{\ddot{D}}_{PW}\right)\}.$$

Proof: 1. Since $S\dot{D}_{PW} \cup S\ddot{D}_{PW} = SD_{PW}$, then $\overline{S\dot{D}_{PW} \cup S\ddot{D}_{PW}} = S\bar{D}_{PW} = S\bar{\dot{D}}_{PW} \cap S\bar{\ddot{D}}_{PW}$ (by De-Morgan laws), and so 2.

Definition (1.20): Let $\dot{H}_{PW}, S\ddot{H}_{PW} \subseteq SD_{PW}$ be sub $-$ symmetric Digraph $- \dot{P}\dot{W}$ and sub $-$ symmetric Digraph $- \ddot{P}\ddot{W}$ of network symmetric Digraph $- PW$ with $V(S\dot{H}_{PW}) = V(S\ddot{H}_{PW}), E(S\dot{H}_{PW}) \cap E(S\ddot{H}_{PW}) = \varphi$, and $S\dot{H}_{PW} \cup S\ddot{H}_{PW} = SD_{PW}$.Then $S\dot{H}_{PW} = (V(SD_{PW}), A^*(S\dot{H}_{PW}), \Phi(S\dot{H}_{PW}))$ where
$A^*(S\dot{H}_{PW}) = A^*(S\ddot{H}_{PW}) \cup \{\bar{n}(a): \bar{n}(a) = max(\dot{P}\dot{W}, \ddot{P}\ddot{W}) - n(a), n(a)\ \in A^*(\ \dot{H}_{PW})\}$ and
$\ddot{H}_{PW} = (V(SD_{PW}), A^*(S\ddot{H}_{PW}), \Phi(S\ddot{H}_{PW}))$where
$A^*(S\ddot{H}_{PW}) = A^*(\ S\dot{H}_{PW}) \cup \{\bar{n}(a): \bar{n}(a) = \max\ (\dot{P}\dot{W}, \ddot{P}\ddot{W}) - n(a), n(a)\ \in A^*(\ \ddot{H}_{PW})\}$

Theorem (1.18): $S\dot{H}_{PW} \cup S\ddot{H}_{PW} = SD_{PW} \cup S\bar{D}_{PW}$.

Proof: By definition (1.3) and (1.9), the result follows.

Definition (1.21): Let $S\dot{Y}_{PW}, S\ddot{Y}_{PW} \subseteq SD_{PW}$ be sub $-$ symmetric Digraph $-\dot{P}\dot{W}$ and sub $-$ symmetric Digraph $-\ddot{P}\ddot{W}$ of network

symmetric Digraph $- PW$with $S\dot{Y}_{PW} = (V(SD_{PW}), A^*(S\dot{Y}_{PW}), \Phi(S\dot{Y}_{PW}))$,

$A^*(S\dot{Y}_{PW}) = \{\bar{n}(\mathrm{a}): \bar{n}(\mathrm{a}) = \dot{P}\dot{W} - n(\mathrm{a})\ \forall v_i, v_j \in V(SD_{PW})\}$ and

$S\ddot{Y}_{PW} = (V(SD_{PW}), A^*(S\dot{Y}_{PW}), \Phi(S\ddot{Y}_{PW}))$,

$A^*(S\ddot{Y}_{PW}) = \{\bar{n}(\mathrm{a}): \bar{n}(\mathrm{a}) = \ddot{P}\ddot{W} - n(\mathrm{a})\ \forall v_i, v_j \in V(SD_{PW})\}$.

Theorem (1.19): $S\dot{D}_{PW} \cup S\dot{Y}_{PW}$and $S\ddot{D}_{PW} \cup S\ddot{Y}_{PW}$are complete symmetric Digraph $-\dot{P}\dot{W}$ and symmetric Digraph-$\ddot{P}\ddot{W}$.

Proof: By definition (1.3) and (1.10)we have

$SD_{PW} \cup S\overline{D}_{PW} = S\dot{D}_{PW} \cup S\dot{Y}_{PW}$ or $S\ddot{D}_{PW} \cup S\ddot{Y}_{PW}$with

$PW = \dot{P}\dot{W}$and $PW < \ddot{P}\ddot{W}$ or $PW = \ddot{P}\ddot{W}$and $PW < \dot{P}\dot{W}$which are completes.

Definition (1.22): Let $S\dot{D}_{PW} = (V, A^*, \Phi) \subseteq SD_{PW}$ be connected sub $-$ symmetric Digraph $-\dot{P}\dot{W}$ with loops and $V(S\dot{D}_{PW}) = V(SD_{PW})$ and let $S\dot{D}^2_{PW} = (V,\ A^*_2, \Phi_2)$ define as-well- as$S\dot{D}_{PW}$ with joint two vertices v_i, v_jnon-adjacency by $n(\mathrm{a}_{ij}) = v_i * v_j$ with $n(\mathrm{a}_{ir})$ and $n(\mathrm{a}_{rj})$ adjacency by v_r, that is, equivalence $1 \leq d(v_i, v_j) \leq 2$, where v_i is adjacent to $v_j$in $S\dot{D}_{PW}$, $d(v_i, v_j)$ is the minimum numbers of vertices except first or last vertex between v_i, v_j. We can continuous define $S\dot{D}^3_{PW} = (V,\ A^*_3, \Phi_3)$, … , then, $S\dot{D}^r_{PW} = (V,\ A^*_r, \Phi_r)$ is called a **Power symmetric Digraph-$\dot{P}\dot{W}$**.

Theorem (1.20): $S\dot{D}^r_{PW} = SD_{PW} iff\ r = maxd(v_i, v_j)$

Proof: Since $S\dot{D}_{PW} = (V, A^*, \Phi) \subseteq SD_{PW}$with the loops, then$S\dot{D}^2_{PW} \subseteq SD_{PW}$, and so on $S\dot{D}^{r-1}_{PW} \subseteq SD_{PW}$, and $S\dot{D}^r_{PW} = SD_{PW} iff\ r = maxd(v_i, v_j)$.

Now, $S\dot{R}_{pW}$, $S\overline{\dot{D}}_{PW}$, $S\overline{\ddot{D}}_{PW} S\dot{H}_{PW}$, $S\ddot{H}_{PW}$, $S\dot{Y}_{PW}$, $S\ddot{Y}_{PW}$, and $S\dot{D}^r_{PW}$ well define connected network symmetric Digraph-$\dot{P}\dot{W}$and symmetric Digraph-$\ddot{P}\ddot{W}$.

Theorem (1.21): $G_{PW} = SD_{PW} \quad iff\ V(G_{PW}) = V(SD_{PW}), and \quad D_{PW} = D^{-1}_{PW}$

Proof: $G_{PW} = SD_{PW} \quad iff \quad SD_{PW} = D_{PW} \cup D^{-1}_{PW}$ satisfy $V(G_{PW}) = V(SD_{PW})$,

$$e = \{v_i, v_j\} = \mathrm{a} = (a_{ij} = (v_i, v_j) \cup (v_j, v_i) = a^{-1}_{ij})\ \forall\ i \neq j, i = j, and$$

$\underset{v_i \in V(G_{PW})}{\deg} v_i = \underset{v_i \in V(SD_{PW})}{\mathrm{indeg}} v_i = \underset{v_i \in V(SD_{PW})}{\mathrm{oudeg}} v_i$, and

$$\underset{n(e_{ij}) \in E^*}{\deg} n(e_{ij}) = \underset{n(\mathrm{a}) \in A^*}{\mathrm{indeg}} n(\mathrm{a}) = \underset{n(\mathrm{a}) \in A^*}{\mathrm{oudeg}} n(\mathrm{a})$$

Moreover $\quad E^* = A^*\ iff\ E^* = (A(D_{PW}) = A(D^{-1}_{PW}))$, and$P_2(V) = V^2 \quad iff \quad G_{PW} = (D_{PW} = D^{-1}_{PW})$.

2. Networks Change Graph-$PW$and Change Symmetric Digraph-PW:

Definition (2.1): Let $G_{PW} = (V, E, \Phi)$ be Graph-*PW*. Then we can define the **change Graph-PW** as $L(G_{PW}) = (V(E^*), E^*(L), L(\Phi))$with $V(E^*) = \{\ n\ (e_{ij}): n\ (e_{ij}) \in E^*(G_{PW})\}$,

$E^*(L) = \{e_{ij} = n(e_{ir})n(e_{rj}): n(e_{ij}) = n(e_{ir}) * n(e_{rj}) = \rho\omega\}$,

$E^*(L) = \{e_{ij} = n(e_{ir})n(e_{rj}): n(e_{ij}) = v_i * v_j = \rho\omega\}$,

$E^*(L) = \{e_{ij} = n(e_{ir})n(e_{rj}): n(e_{ij}) = v_r * v_r = \rho\omega\}$ or

$E^*(L) = \{e_{ij} = n(e_{ir})n(e_{rj}): n(e_{ij}) = v_r = \rho\omega\}$, that is,

the set $n\ (e_{ij})$-edges of $E^*\ (G_{PW})$ is vertices in $L(G_{PW})$, and the e_{ij} in $E^*(L)$ if and only if $n(e_{ir})\ and\ n(e_{rj})$ are adjacency of vertex v_r ,

$$L(\Phi): E^*(L) \to P_2(V(E^*)), \text{and}$$

$PW = max\rho\omega = maxn(e_{ij}) * n(e_{rj}), v_i * v_j , v_r * v_r or\ v_r = \rho\omega$.

Moreover, we can be found $L^2(G_{PW}) = L(L(G_{PW}))$,..., $L^n(G_{PW}) = L(L^{n-1}(G_{PW}))$.
Applied all definitions in the definition(1.1) and definition (1.2) in the definition (2.1).

Definition (2.2): The **complement** of network change Graph-*PW*
$L(G_{PW}) = \big(V(E^*), E^*(L), L(\Phi)\big)$ is $\bar{L}(G_{PW}) = (V(E^*), \overline{E^*}(\bar{L}), \bar{L}(\overline{\Phi}))$ with
$\overline{E^*}(L) = \{e_{ij}: \bar{n}(e_{ij}) = PW - n(e_{ij}), i \neq j \ or \ i = j\}$
$\bar{L}(\overline{\Phi}): \overline{E^*}(\bar{L}) \to P_2\big(V(E^*)\big)$, and

$$PW = max\, \bar{n}(e_{ij}).$$

Moreover, $L(G_{PW}) \cup \overline{E^*}(L) = K_{|V(E^*)|} - PW$.

Definition (2.3): Let $G_{PW} = (V, E^*, \Phi)$ be network regular Graph-*PW.* Then we can define the network change regular Graph-*PW as* $L(G_{PW}) = \big(V(E^*), E^*(L), \Phi(L)\big)$ *with*
$V(E^*) = \{PW: PW \in E^*(G_{PW})\}$,

$$E^*(L) = \left\{e_{ij} : n(e_{ij}) = \frac{|V|^2(|V|-1)}{2} PW * PW = \rho\omega\right\},$$

$\Phi(L): E^*(L) \to P_2\big(V(E^*)\big)$, and $PW = max\rho\omega$.

Definition (2.4): Applied definitions (1.5) and (1.6) in the network change Graph-*PW*.

Theorem (2.1): Applied theorems (1.1),(1.2),(1.3) and (1.4) in the network change Graph-*PW*.

Clear that if $G_{PW} = (V, E^*, \Phi)$ is connected Graph-*PW.* Then $L(G_{PW}) = \big(V(E^*), E^*(L), L(\Phi)\big)$ is connected change Graph-*PW*, if $G_{PW} = (V, E^*, \Phi)$ has a soiled vertex, then $\bar{G}_{PW} = (V, \overline{E^*}, \overline{\Phi})$ or $G_{PW} \cup \bar{G}_{PW}$ are connected, so $L(\bar{G}_{PW})$ and $L(G_{PW} \cup \bar{G}_{PW})$,moreover can be looking the change Graph-*PW* $L(G_{PW} \cup \bar{G}_{PW})$has only one vertex with the loops. If $G_{PW} = (V, E^*, \Phi)$ has a soiled vertex, then
$L(G_{PW}) = \big(V(E^*), E^*(L), L(\Phi)\big)$ is connected .

Definition (2.5): The definitions (1.7) define similar of definition (2.3) in the network change Graph-*PW*.

Definition (2.6): Applied the definitions (1.8), (1.9),(1.10) and (1.11) in the network change Graph-*PW*.

Theorem (2.2): Applied theorems (1.6), (1.7),(1.8), (1.9) and (1.10) in the network change Graph-*PW*.
Now, let $\dot{G}_{PW}, \ddot{G}_{PW} \subseteq G_{PW}$ *be Graph* $- \dot{P}\dot{W}$ *and Graph* $- \ddot{P}\ddot{W}$ *with*
$V(\dot{G}_{PW}) = V(\ddot{G}_{PW}), E(\dot{G}_{PW}) \cap E(\ddot{G}_{PW}) = \varphi$, *and* $\dot{G}_{PW} \cup \ddot{G}_{PW} = G_{PW}$.
Clear that $\dot{G}_{PW}$ is $L(\dot{G}_{PW})$, and $L^2(\dot{G}_{PW}) = L(L(\dot{G}_{PW}))$,..., $L^n(\dot{G}_{PW}) = L(L^{n-1}(\dot{G}_{PW}))$ and $\ddot{G}_{PW}$ is $L(\ddot{G}_{PW})$, and $L^2(\ddot{G}_{PW}) = L(L(\ddot{G}_{PW}))$,..., $L^n(\ddot{G}_{PW}) = L(L^{n-1}(\ddot{G}_{PW}))$.

Theorem (2.3): $\bar{\dot{G}}_{PW}, \bar{\ddot{G}}_{PW}, \dot{H}_{PW}$ and $\ddot{H}_{PW}$ have network change Graph-*PW* and

$$L\left(\bar{\dot{G}}_{PW}\right), L^2\left(\bar{\dot{G}}_{PW}\right) = L\left(L\left(\bar{\dot{G}}_{PW}\right)\right), \dots, L^n\left(\bar{\dot{G}}_{PW}\right) = L(L^{n-1}\left(\bar{\dot{G}}_{PW}\right),$$

$L\left(\bar{\ddot{G}}_{PW}\right), L^2\left(\bar{\ddot{G}}_{PW}\right) = L\left(L\left(\bar{\ddot{G}}_{PW}\right)\right), \dots, L^n\left(\bar{\ddot{G}}_{PW}\right) = L(L^{n-1}\left(\bar{\ddot{G}}_{PW}\right))$,
$L(\dot{H}_{PW}), L^2(\dot{H}_{PW}) = L(L(\dot{H}_{PW}))$,..., $L^n(\dot{H}_{PW}) = L(L^{n-1}(\dot{H}_{PW}))$ and
$L(\ddot{H}_{PW}), L^2(\ddot{H}_{PW}) = L(L(\ddot{H}_{PW}))$,..., $L^n(\ddot{H}_{PW}) = L(L^{n-1}(\ddot{H}_{PW}))$.

Proof: By the definition (2.1) the result follows.

Theorem (2.4): Every $\dot{G}^r_{PW} = (V, E^*_r, \Phi_r)$ there is $L(\dot{G}^r_{PW}), r = 1,2,3, \dots, maxd(v_i, v_j)$.
Moreover, $L^2(\dot{G}^r_{PW}) = L(L\, \dot{G}^r_{PW}), \dots, L^n(\dot{G}^r_{PW}) = L^{n-1}(\dot{G}^r_{PW})$.

Proof: By theorem (1.10) and the definition (2.1) the result follows.

Definition (2.7): Let $SD_{PW} = (V, A^*, \Phi)$ be Symmetric Digraph-*PW.* Then we can define the change symmetric Digraph-*PW is* $L(SD_{PW}) = \big(V(A^*), A^*(L), L(\Phi)\big)$with
$V(A^*) = \{\, n(\,\mathrm{a}) \colon n\,(\mathrm{a}) \in A^*(SD_{PW})\}$,
$A^*(L) = \{\mathrm{a}_{ij} = n(\mathrm{a}_{ir})\, n(\mathrm{a}_{rj}) \colon n(\mathrm{a}_{ij}) = n(\mathrm{a}_{ir}) * n(\mathrm{a}_{rj}) = \rho\omega\}$,
$A^*(L) = \{\mathrm{a}_{ij} = n(\mathrm{a}_{ir})\, n(\mathrm{a}_{rj}) \colon n(\mathrm{a}_{ij}) = \ v_i * v_j = \rho\omega\}$,
$A^*(L) = \{\mathrm{a}_{ij} = n(\mathrm{a}_{ir})\, n(\mathrm{a}_{rj}) \colon n(\mathrm{a}_{ij}) = \ v_r * v_r = \rho\omega\}$ or
$A^*(L) = \{\mathrm{a}_{ij} = n(\mathrm{a}_{ir})\, n(\mathrm{a}_{rj}) \colon n(\mathrm{a}_{ij}) = \ v_r \ = \rho\omega\}$,
that is, the symmetric set $n\,(\mathrm{a}_{ij})$-arcs of $A^*\,(SD_{PW})$ is vertices in $L(SD_{PW})$, and the a_{ij} in $A^*(L)$ if and only if $n(\mathrm{a}_{ir})$ and $n(\mathrm{a}_{rj})$ are adjacency of vertex v_r ,

$$L(\Phi) \colon A^*(L) \to \ V^2(A^*), \text{and}$$

$PW = max\rho\omega = max\; n(\mathrm{a}_{ij}) * n(\mathrm{a}_{rj}), v_i * v_j \,, v_r * v_r or\; v_r \ = \rho\omega.$

Moreover, we can be found $L^2(SD_{PW}) = L(L(SD_{PW})\,),\ldots, L^n\,(SD_{PW}) = L(L^{n-1}(SD_{PW}))$.
Applied all definitions in the definition (1.12) and definition (1.13) in the definition (2.7).

Definition (2.8): The complement of change symmetric Digraph-*PW*
$\mathrm{L}(SD_{PW}) = \big(V(A^*), A^*(L), L(\Phi)\big)$ is $\bar{L}(SD_{PW}) = (V(A^*), \overline{A^*}(\bar{\mathrm{L}}), \bar{\mathrm{L}}\,(\overline{\Phi}\,))$with
$\overline{A^*}(\bar{\mathrm{L}}) = \{\mathrm{a}_{ij} \colon \bar{n}(\mathrm{a}_{ij}) = PW - \ n(\mathrm{a}_{ij})\,, i \neq j \; or \; i = j\}$
$\bar{\mathrm{L}}(\overline{\Phi}) \colon \overline{A^*}(\bar{\mathrm{L}}) \ \to V^2(A^*)$,, and

$$PW = max\; \bar{n}(\mathrm{a}_{ij}) = max\rho\omega.$$

Moreover, $L(SD_{PW}) \cup \overline{A^*}(L) = K_{|V(A^*)|} - PW$

Definition (2.9): Let $SD_{PW} = (V, A^*, \Phi)$ be regular symmetric Digraph-*PW.* Then we can define the change regular symmetric Digraph-*PW* is
$\mathrm{L}(G_{PW}) = (V(E^*), E^*(\mathrm{L}), \Phi(\mathrm{L})) with\, V(E^*) = \{\, PW \colon \ PW \in E^*(G_{PW})\}$,

$$A^*(\,\mathrm{L}) = \left\{\mathrm{a}_{ij} \, \colon n(\mathrm{a}_{ij}) = \frac{|V|^2(|V|-1)}{2} PW * PW = \rho\omega\right\}$$

$$\Phi(\mathrm{L}) \colon A^*(\mathrm{L}) \to P_2\big(V(E^*)\big), \text{and } PW = max\rho\omega$$

Definition (2.10): Applied definitions (1.14) and (1.15) in the network change symmetric Digraph-*PW*.

Theorem (2.5): Applied theorems (1.12), (1.3), (1.14) and (1.15) in the network change symmetric Digraph-*PW*.
Clear that if $SD_{PW} = (V, A^*, \Phi)$ is connected symmetric Digraph-*PW,* then $L(SD_{PW}) = \big(V(A^*), A^*(L), L(\Phi)\big)$ is connected change symmetric Digraph-*PW*, if $SD_{PW} = (V, A^*, \Phi)$ has a soiled vertex, then $S\bar{D}_{PW} = (V, \overline{E^*}, \bar{\Phi})$ or $SD_{PW} \cup S\bar{D}_{PW}$ are connected, so $L(S\bar{D}_{PW})$ and $\mathrm{L}(\,SD_{PW} \cup S\bar{D}_{PW})$,moreover can be looking the change symmetric Digraph-*PW* $\mathrm{L}(\,SD_{PW} \cup S\bar{D}_{PW})$has only one vertex with the loops. If $SD_{PW} = (V, A^*, \Phi)$ has a soiled vertex, then $L(SD_{PW}) = \big(V(A^*), A^*(L), L(\Phi)\big)$ is connected .

Definition (2.11): The definition (1.16) define similar of definition (2.9) in the network change symmetric Digraph-*PW*.

Definition (2.12): Applied the definitions (1.17),,(1.18),(1.19) and (1.20) in the network change symmetric Digraph-*PW*.

Theorem (2.6): Applied theorems (1.17),(1.18),(1.19), and (1.20) in the network change symmetric Digraph-*PW*.
Now, let $\dot{G}_{PW}, \ddot{G}_{PW} \subseteq G_{PW}\; be\; Graph - \dot{P}\dot{W}\; and\; Graph - \ddot{P}\ddot{W}$ with
$V(\dot{G}_{PW}) = V(\ddot{G}_{PW}), E(\,\dot{G}_{PW}) \cap E(\ddot{G}_{PW}) = \varphi, and\; \dot{G}_{PW} \cup \ddot{G}_{PW} = G_{PW}.$

Clear that $\dot{G}_{PW}$ is $L(\dot{G}_{PW})$, and $L^2(\dot{G}_{PW}) = L(L(\dot{G}_{PW}))$,..., $L^n(\dot{G}_{PW}) = L(L^{n-1}(\dot{G}_{PW}))$ and $\ddot{G}_{PW}$ is $L(\ddot{G}_{PW})$, and $L^2(\ddot{G}_{PW}) = L(L(\ddot{G}_{PW}))$,..., $L^n(\ddot{G}_{PW}) = L(L^{n-1}(\ddot{G}_{PW}))$

Theorem (2.7): $S\bar{D}_{PW}, S\bar{\bar{D}}_{PW}, S\dot{H}_{PW}$ and $S\ddot{H}_{PW}$ have network change symmetric Digraph-*PW* and

$$L\left(S\bar{D}_{PW}\right), L^2\left(S\bar{D}_{PW}\right) = L\left(L\left(S\bar{D}_{PW}\right)\right), \dots, L^n\left(S\bar{D}_{PW}\right) = L\left(L^{n-1}\left(S\bar{D}_{PW}\right)\right),$$

$L\left(S\bar{\bar{D}}_{PW}\right), L^2\left(S\bar{\bar{D}}_{PW}\right) = L\left(L\left(S\bar{\bar{D}}_{PW}\right)\right), \dots, L^n\left(S\bar{\bar{D}}_{PW}\right) = L\left(L^{n-1}\left(S\bar{\bar{D}}_{PW}\right)\right)$,

$L(S\dot{H}_{PW})$, $L^2(S\dot{H}_{PW}) = L(L(S\dot{H}_{PW}))$,..., $L^n(S\dot{H}_{PW}) = L(L^{n-1}(S\dot{H}_{PW}))$ and

$L(S\ddot{H}_{PW})$, $L^2(S\ddot{H}_{PW}) = L(L(S\ddot{H}_{PW}))$,..., $L^n(S\ddot{H}_{PW}) = L(L^{n-1}(S\ddot{H}_{PW}))$.

Proof: By the definition (2.7) the result follows.

Theorem (2.8): Every $S\dot{D}^r_{PW} = (V, A^*_r, \Phi_r)$ there is $L\left(S\dot{D}^r_{PW}\right), r = 1,2,3, \dots, max d(v_i, v_j)$.
Moreover, $L^2\left(S\dot{D}^r_{PW}\right) = L\left(L\, S\dot{D}^r_{PW}\right), \dots, L^n\left(S\dot{D}^r_{PW}\right) = L^{n-1}(S\dot{D}^r_{PW})$.

Proof: By theorem (1.20), by the definition (2.7) the result follows.

3. CONCLUSION

In this paper, we determined some new Networks. Furthermore, several theorems and results of these networks have been studied. In the future, we are interested in designing some new networks and then studying their topological indices which will be quite helpful in understanding their underlying topologies.

4. COMPLIANCE WITH ETHICAL STANDARDS

4.1 AUTHORS' CONTRIBUTIONS

A. Alameri conceived of the presented idea, M. Alsharafi prepared the initial manuscript, W. A.M. Saeed developed the theory, A. Ghallab performed the computations, W. Yousef verified the analytical methods and A. Modabish supervised the findings of this work. All authors jointly worked on the results and they read and approved the final manuscript.

4. 2 AVAILABILITY OF DATA AND MATERIAL

No data were used to support this study.

4.3 DECLARATION OF COMPETING INTEREST

The authors declare that they have no conflict of interest.

4. 4 FUNDING STATEMENT

This research did not receive any specific grant.

REFERENCES

1) L. Rumberg, C. Gebauer, H. Ehlert, U. Lüdtke, & J. Ostermann. "Improving Phonetic Transcriptions of Children's Speech by Pronunciation Modelling with Constrained CTC-Decoding", Proc. Interspeech, pp. 1357-1361, 2022.
2) J. Clark. and D. A. Han Holton. "An Introduction to Graphs" in "A first look at Graph theory", 1st ed, pp. 1-47, 1995.
3) M. Alsharafi, Y. Zeren, A. Alameri. "The Second Hyper-Zagreb Index of Complement Graphs and Its Applications of Some Nano Structures." Asian Journal of Probability and Statistics, 15(4): pp. 54-75, 2021.
4) I. Gutman, B. Ruščić, N. Trinajstić, C.F. Wilcox Jr, Graph theory and molecular orbitals. XII. Acyclic polyenes, J. Chem. Phys. 62 (9); pp. 3399–3405, 1975.
5) A.R. Ashrafi, T. Došlić, A. Hamzeh, The Zagreb coindices of graph operations, Discret Appl. Math. 158 (15); pp.1571–1578, 2010.
6) G. H. Shirdel, H. Rezapour, and A. M. Sayadi, "&e hyper Zagreb index of graph operations," Iranian Journal of Mathematical Chemistry, vol. 4, no. 2, pp. 213–220, 2013.

7) P. S. Ranjini, V. Lokesha, and A. Usha, “Relation between phenylene and hexagonal squeeze using harmonic index,” International Journal of Graph 5eory, vol. 1, no. 4, pp. 116–121, 2013.
8) M. Veylaki, M. J. Nikmehr, and H. A. Tavallaee, “&e third and hyper Zagreb coindices of some graph operations,” Journal of Applied Mathematics and Computing, vol. 50, no. 12, 2016, pp. 315–325.
9) W. Gao, M. R. Farahani, M. K. Siddiqui, and M. K. Jamil, ``On the first and second zagreb and first and second hyper-zagreb indices of carbon nanocones CNCk[n],'' J. Comput. Theor. Nanosci., vol. 13, no. 10, 2016, pp. 7475-7482.
10) B. Furtula and I. Gutman, “A forgotten topological index, ”Journal of Mathematical Chemistry, vol. 53, no. 4, 2015, pp. 1184–1190.
11) . De, S. Nayeem and A. Pal, The F-coindex of some graph operations, SpringerPlus 5.1 (2016): 221.
12) A. Alameri, N. Al-Naggar, M. Al-Rumaima, M. Alsharafi, Y-index of some graph operations, Int. J. Appl. Eng. Res. 15 (2) (2020) 173e179.
13) A. Alameri, M. Al-Rumaima, and M. Almazah, Y-coindex of graph operations and its applications of molecular descriptors, J. Mol. Struct., vol. 1221, Dec. 2020, Art. no. 128754.
14) A. Alameri, Second Hyper-Zagreb index of Titania Nanotubes and their Applications, IEEE Access. 9 (2021), 9567-9571.
15) A. Modabish, A. Alameri, M. S. Gumaan, M. Alsharafi, The second Hyper-Zagreb index of graph operations. J. Math. Comput. Sci., 11(2), (2021) 1455-1469.
16) H. Ahmed, A. Alwardi, M. R. Salestina, and D. S. Nandappa, “Domination, c-domination, topological indices and ϕp-polynomial of some chemical structures applied for the treatment of COVID-19 patients,” Biointerface Research in Applied Chemistry, pp. 13290–13302, 2021.
17) M. S. Alsharafi, M. M. Shubatah, M. Q. Alameri, On the Hyper-Zagreb coindex of some graphs. J. Math. Comput. Sci., 10(5), (2020) 1875-1890.
18) M. S. Alsharafi, M. M. Shubatah, M. Q. Alameri, The hyper-Zagreb index of some complement graphs. Advances in Mathematics: Scientific Journal, 9(6), (2020) 3631-3642.
19) A. Ayache, A. Alameri, A. Ghallab, A. Modabish, Wiener polynomial and Wiener index of conical graphs, Sylwan Journal, 164(3), (2020), 107-116.
20) M. Alsharafi, M. Shubatah, A. Alameri, The forgotten index of complement graph operations and its applications of molecular graph, Open Journal of Discrete Applied Mathematics. 3(3), (2020), 53-61.
21) A. Alameri, New Binary operations on Graphs, Journal of Science and Technology, (2016), 1607- 2073.
22) M. Alsharafi, M. Shubatah, A. Alameri, The First and Second Zagreb Index of Complement Graph and Its Applications of Molecular Graph, Asian Journal of Probability and Statistics. 8(3), (2020), 15-30.
23) A. Alameri, M. Shubatah, M. Alsharafi, Zagreb indices, Hyper-Zagreb indices and Redefined Zagreb indices of conical graph, Advances in Mathematics: Scientific Journal, 9(6), (2020), 3631-3642.
24) M. Alsharafi, M. Shubatah, A. Alameri, On the Hyper-Zagreb coindex of some Graphs, J. Math. Comput. Sci. 10(5), (2020), 1875-1890.
25) A. Alameri, A. Modabish, A. Ayache, New Product Operation on Graphs and its Properties, International Mathematical Forum, Vol. 11, 2016, no. 8, 357 - 368.
26) M. Alsharafi, M. Shubatah, A. Alameri, The hyper-Zagreb index of some complement graphs, Advances in Mathematics: Scientific Journal, 9(6), (2020), 3631-3642.
27) M. Bilal, M. K. Jamil, M. Waheed, A. Alameri, Three Topological Indices of Two New Variants of Graph Products, Mathematical Problems in Engineering, V (2021), ID 7724177, 9 pages, 19 November 2021.
28) A. Alameri, M. Alsharafi, E. Ali, M. S. Gumaan, A note on Topological indices and coindices of disjunction and symmetric difference of graphs. Discret. Appl. Math. 2021, 304, 230-235, https://doi.org/10.1016/j.dam.2021.07.037.
29) A. Ayache, A. Alameri, M. Alsharafi, H. Ahmed. The Second Hyper Zagreb Coindex of Chemical Graphs and Some Applications, Journal of Chemistry; 2021. ID 3687533, 8 pages.