

Correlation between Relational Understanding and Cognitive Style on Mathematical Problem-Solving Ability



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ABSTRACT: Problem-solving ability is needed in mathematics learning. This study aimed to find out about the correlation between relational understanding and cognitive style on the problem-solving ability of students. Relational understanding was taken into account because this understanding is related to a person's ability to use mathematical procedures obtained by correlating various mathematical concepts that are relevant in problem-solving and to understand why such procedures can be used. Cognitive style was also considered because this style is related to how students think and understand, including how they solve problems. The study used multiple correlation analysis with independent variables of relational understanding (X1) and cognitive style (X1), while the dependent variable was problem-solving ability (Y). The samples of this study were 22 High School students in Malang, selected randomly. The multiple correlation analysis resulted in $R = 0,965$, indicating that there is a significant correlation between relational understanding and cognitive style on problem-solving ability. In addition, the result of F test also showed that the multiple correlation coefficient was significant.

KEYWORDS: Relational understanding; Cognitive style; Problem-solving ability

INTRODUCTION

A problem is an obstacle or issue to be solved. Most mathematicians argue that problems are questions that students shall answer or respond to, but not all questions will become problems for students (Shadiq, 2003). A question will become a problem if the question contains a challenge that is not solved by any routine procedures that students already know. Therefore, such question will become a problem for students who do not know the routine procedures or find a challenge in solving it, but the same question will only become an ordinary question for students who already know the procedure.

According to Reys (in Zhu, 2007), "problem solving is the foundation of much mathematical activity". Problem solving is a fundamental component in learning and gaining knowledge (Ayllon, et al, 2016). According to Polya, problem solving processes consist of (1) understanding the problem, (2) devising a plan, (3) carrying out the planning, and (4) re-examining (Ayllon, et al, 2016). The steps that can be selected in solving are guessing and checking, making an orderly list, eliminating possibilities, using symmetry, considering special cases, using direct reasoning, solving equations, looking for patterns, drawing pictures, solving simpler problems, using a model, walking backwards, using a formula and becoming ingenious (Victor, 2015).

Problem solving is taught and explicitly becomes one mathematics learning objective contained in mathematics curriculum. This is in line with the mathematics learning objectives stated by BSNP (2006). In addition, NCTM (2000) states that problem solving is one of the standards that shall be mastered by students in mathematics learning. In addition, problem solving ability is seen not only as a mathematics learning objective but also as the main instrument to measure the performance of students in mathematics learning (Eviyanti, et al, 2017). In fact, mathematics and problem solving are two things that are integral.

Problem solving is needed in mathematics learning. This, according to Pehkonen (1997), is because problem solving has various benefits, i.e.: (1) developing general cognitive skills, (2) boosting creativity, (3) problem solving is a part of mathematics application processes, and (4) motivating students to learn mathematics. In addition, problem solving could improve students' higher-order thinking skills (Abdullah, et al, 2015; Kortesi & Georgieva, 2015; Ersoy, 2016). Based on this explanation, problem solving is one of the mathematics learning objectives that could encourage and train students' higher-order thinking skills. Therefore, teachers shall pay attention to any factors that influence or are related to problem solving abilities. By knowing

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these factors, teachers are expected to formulate a learning design that is able to provide a positive relationship with problem solving abilities.

One of the factors considered to have a positive relationship with students' problem-solving ability is students' understanding. Understanding, in Revised Bloom's Taxonomy, is a dimension higher than remembering which consists of interpreting, giving examples, classifying, summarizing, concluding, comparing and explaining (Krathwohl, 2002). Further Skemp (1987) divided understanding into three, namely instrumental understanding, relational understanding, and logical understanding.

This understanding specifically refers to relational understanding. Relational understanding is students' understanding to find the procedures to be used, based on reasoning and ability to connect to mathematical concepts (Anwar, 2016). Relational understanding is a rich and interconnected network of concepts (Olivia, et al, 2013). This understanding is the ability to re-explain the concepts that have been studied, analyze (classify), apply the concepts in an algorithm, and correlate one concept to another so that students are able to find reasoning or rationality of an answer (Mustaghfirin, 2014). This is in line with an argument of Skemp (1987) that "relational understanding is described as knowing both what to do and why", i.e. a person's ability to use mathematical procedures obtained by correlating various mathematical concepts that are relevant in problem-solving and to understand why such procedures can be used. Yazidah (2018) in a previous research stated that groups of students with high academic achievement and students with middle academic achievement had almost the same relational understanding abilities.

Relational understanding is crucial in mathematics learning. In addition, improving relational understanding will then improve meaningful learning for students (Anwar, 2016). Relational understanding is important for meaningful mathematics learning because teachers certainly expect that students' understanding is not limited to the understanding to only connect (Mustaghfirin, 2014). Besides, relational understanding contains a scheme or structure that can be used to solve more problems (Mustaghfirin, 2014). Therefore, relational understanding can be said to play a role in problem solving.

In addition to relational understanding, another factor considered is cognitive style. Cognitive style is a psychological term to describe the way individuals think, understand and memorize information (Lusiana, 2017). Cognitive style is an individual characteristic in thinking, feeling, memorizing, solving problems, and making decisions (Park, 1996: 639). Cognitive style can be divided into two, namely field independent (FI) and field dependent (FD) cognitive styles.

Further, each of these cognitive styles has its own characteristics. Individuals with field dependent cognitive style are those who tend to think globally, view an object and its environment as a unity, have social orientation, prefer a structured environment, follow existing objectives, as well as prioritize extrinsic motivation and reinforcement. On the other hand, individuals with independent field cognitive style are those who tend to view objects as discrete and separate parts of the environment, able to analyze to separate stimuli from the context, able to restructure, have impersonal orientation, design their own objectives, and work with intrinsic motivation and reinforcement. In other words, cognitive style is related to how students think and understand, including their problem solving.

This study analyzed the correlation between these two factors, both relational understanding and cognitive style, with problem solving ability. A strong correlation between relational understanding and cognitive style with problem solving ability can be used as a consideration in designing a more meaningful mathematics learning for students. This study was limited to only relational understanding and cognitive style as the independent variables and problem-solving ability as the dependent variable.

METHODS AND MATERIALS

Scope of Research

The research instrument was test questions which were limited to combinatorics to measure students' relational understanding and problem-solving ability. Combinatorics was selected because this material is one of the basic concepts that students shall master and this is highly relevant to everyday life. In addition, the research instrument was also in the form of GEFT (Group Embedded Figure Test) to find out about students' cognitive style, whether the students had field dependent (FD) or field independent (FI) styles.

Research design

This was a correlational research with a quantitative approach. The correlational technique used was multiple correlation with Pearson Product Moment that matched the ratio data. The multiple correlation in this study was the correlation between two independent variables and one dependent variable (Sugiyono, 2002). The independent variables in this study were relational understanding (X_1) and cognitive style (X_2). The dependent variable was problem-solving ability (Y).

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Research Procedure

The population in this study was high school students in Malang, while the samples consisted of 22 students who were randomly selected. The samples were selected using simple random sampling technique.

Data on problem solving ability scores (Y), relational understanding (X₁) and cognitive style (X₂) will be analyzed correlation between problem-solving ability (Y) and relational understanding (X₁), correlation between problem-solving ability (Y) and cognitive style (X₂), correlation between relational understanding (X₁) and cognitive style (X₂), correlation between relational understanding (X₁) and cognitive style (X₂) with problem-solving ability (Y).

Data Collection and Data Analysis

Multiple correlation is number that indicates the direction and strength of the correlation between two or more variables simultaneously with other variables (Sugiyono, 2002). The correlation between two or more variables is stated to be positive when an increased value of a variable could increase the value of another variable, and vice versa, when a decreased value of a variable could decrease the value of another variable. To calculate multiple correlation, simple correlation was first calculated using Pearson Product Moment correlation. The simplest formula that can be used to calculate the correlation coefficient is as follows,

$$r_{xy} = \frac{\sum xy}{(\sum x^2)(\sum y^2)} \quad (1)$$

where r_{xy} is the correlation between variable x and y , $x = x_i - \bar{x}$ with $y = y_i - \bar{y}$. Next, multiple correlation was calculated using the following formula.

$$R_{yx_1x_2} = \sqrt{\frac{r_{yx_1}^2 + r_{yx_2}^2 - 2r_{yx_1}r_{yx_2}r_{x_1x_2}}{1 - r_{x_1x_2}^2}} \quad (2)$$

Interpretation of correlation coefficient adapted from Sugiyono (2002) is presented in the following table.

Table 1. Interpretation of Correlation Coefficient

Coefficient Interval	Correlation Level
$0 \leq r < 0.2$	Very low
$0.2 \leq r < 0.4$	Low
$0.4 \leq r < 0.6$	Moderate
$0.6 \leq r < 0.8$	Strong
$0.8 \leq r \leq 1$	Very strong

Next, F test was done to test the significance on the multiple correlation coefficient.

$$F_{count} = \frac{R^2/k}{(1 - R^2)/(n - k - 1)}$$

with R is multiple correlation coefficient, k is the number of independent variables and n is the number of samples. If F_{count} is greater than F_{table} then H_0 is rejected and H_1 is accepted, meaning that the multiple correlation coefficient is significant.

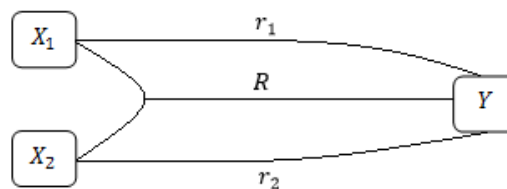


Figure 1. Multiple Correlation between X₁ and X₂ with Y

Based on this diagram, r_1 shows the correlation between relational understanding and problem-solving ability, r_2 shows the correlation between cognitive style and problem-solving ability, and R shows the correlation between relational understanding and cognitive style with problem-solving ability.

RESULTS AND DISCUSSIONS

Results

Problem-solving ability refers to Polya, but this study did not consider the review stage. Therefore, the problem-solving ability indicators were (1) students are able to understand the problem, (2) students are able to devise a plan, and (3) students

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are able to carry out the plan. The combinatorics problem given was "a car number plate of Malang area begins with letter N, followed by four numbers, and ends with two letters. How many number plates can be formed if the four digit numbers form even numbers and contain no same numbers?". The problem-solving ability was presented using a scale of 0 to 100. The guidelines for the scoring of students' problem-solving ability are presented in Table 2.

Table 2. Guidelines for Scoring of Students' Problem-Solving Ability

Problem Indicators	Solving	Description	Scores
Understand problems	the	Interpret the problems correctly	2
		Incorrectly interpret some of the problems	1
		Misinterpret the problems	0
Devise a plan		Devise a problem-solving plan correctly and completely	4
		Devise a problem-solving plan correctly but incompletely	3
		Devise a problem-solving plan that is less relevant	2
		Devise a problem-solving plan that is not relevant	1
		Do not devise a problem-solving plan	0
Carry out the plan		Carry out the problem-solving procedure correctly and obtain correct results	2
		Carry out the problem-solving procedure correctly but obtain incorrect results	1
		Carry out no problem-solving procedure	0

Problem solving ability in this study was the dependent variable, whose correlation with students' relational understanding and cognitive style was analyzed. Students' scores in terms of their problem solving ability were calculated using the following formula.

$$y = \frac{\text{score obtained by students}}{8} \times 100$$

Relational understanding was one of the independent variables whose correlation with problem solving ability was analyzed in this study. The indicators of relational understanding referred to combinatorics problems given to the students. In general, the indicators of relational understanding consisted of: (1) correlating various mathematical concepts, (2) applying concepts in various mathematical representation, and (3) classifying objects based on whether or not the requirements for the concept are met. These three indicators were further explained based on the given combinatorics problem. The descriptions of each indicator are presented in the following table.

Table 3. Description of Relational Understanding Indicators

Indicator	Description
Correlating various mathematical concepts	Able to correlate the concepts of even numbers
Applying concepts in various mathematical representation	Able to transfer the problem into four boxes for numbers and two boxes for letter
Classifying objects based on whether or not the requirements for the concept are met	<ol style="list-style-type: none"> Able to determine the correct enumeration rules to solve the problem based on what is known and asked Able to place even numbers in the last box as the first box to be filled out Able to divide the solution into two cases (if number 0 fills in the last box and if number 0 does not fill in the last box) Able to determine many choices of number in each box

The descriptions of each indicator were eventually used to measure the students' relational understanding. The following table describes the scoring of the students' relational understanding.

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Table 4. Guideline for Scoring of Students' Relational Understanding

Description of Indicator	Description	Score
Able to determine the correct enumeration rules to solve the problem based on what is known and asked	Able to determine the appropriate enumeration rules and explain the reasons of determining the rules based on what is known and asked	2
	Able to determine the correct enumeration rules but unable to explain the reasons of determining the rules based on what is known and asked	1
	Unable to determine the correct enumeration rules and unable to explain the reasons of determining the rules based on what is known and asked	0
Able to transfer the problem into four boxes for numbers and two boxes for letter	Able to transfer the problem into four boxes for numbers and two boxes for letter	2
	Able to transfer the problem into four boxes for numbers only, but neglecting the letters	1
	Unable to transfer the problem into four boxes for numbers and two boxes for letter	0
Able to correlate the concepts of even numbers	Able to explain the definition of even numbers and correctly mention numbers belonging to even numbers, i.e. 0, 2, 4, 6 and 8	2
	Able to explain the definition of even numbers and but incorrectly mention numbers belonging to even numbers, i.e. 0, 2, 4, 6 and 8	1
	Unable to explain the definition of even numbers and unable to mention numbers belonging to even numbers, i.e. 0, 2, 4, 6 and 8	0
Able to place even numbers in the last box as the first box to be filled out	Able to place even numbers in the last box as the first box to be filled out	1
	Unable to place even numbers in the last box as the first box to be filled out	0
Able to divide the solution into two cases (if number 0 fills in the last box and if number 0 does not fill in the last box)	Able to divide the solution into two cases	1
	Unable to divide the solution into two cases	0
Able to determine many choices of number in each box	Able to determine many choices of number in each box and explain the reason behind each choice of number	2
	Able to determine many choices of number in each box but unable to explain the reason behind each choice of number	1
	Unable to determine many choices of number in each box	0

Each of the students' relational understanding scores was calculated using the following formula.

$$x = \frac{\text{score obtained by students}}{10} \times 100$$

In addition to relational understanding, another independent variable analyzed in this study was cognitive style. Cognitive style scores referred to the GEFT questions given to the students. The minimum score was 0 and the maximum was 18. The score ranging from 0 to 9 showed FD cognitive style, while the score from 10 to 18 showed FI cognitive style. The GEFT test score was then converted into a scale of 0 to 100 using the following formula.

$$x = \frac{\text{score obtained by students}}{18} \times 100$$

Data of problem-solving ability score (Y), relational understanding (X_1) and cognitive style (X_2) are presented in the following table.

Correlation between Problem-Solving Ability (Y) and Relational Understanding (X_1)

To determine the correlation between problem-solving ability and relational understanding, correlation analysis was performed using formula (1). The research showed that there is a significant correlation between relational understanding and

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problem-solving ability, evident from the calculation which resulted in $r_{yx_1} = 0.962$. This shows that there is a strong correlation between relational understanding and problem-solving ability. In addition, the correlation is positive, so the better the students' relational understanding, the better their problem-solving ability.

Correlation between Problem-Solving Ability (Y) and Cognitive Style (X₂)

To determine the correlation between problem-solving ability and cognitive style, correlation analysis was performed using formula (1). The research showed that there is a significant correlation between cognitive style and problem-solving ability, evident from the calculation which resulted in $r_{yx_2} = 0.861$. This shows that there is a strong correlation between cognitive style and problem-solving ability. In addition, the correlation is positive, so the better the GEFT test score, the better the students' problem-solving ability. Higher test scores indicate that students tend to have FI cognitive style.

Correlation between Relational Understanding (X₁) and Cognitive Style (X₂)

To determine the correlation between relational understanding and cognitive style, correlation analysis was performed using formula (1). The research showed that there is a significant correlation between cognitive style and relational understanding, evident from the calculation which resulted in $r_{x_1x_2} = 0.923$. This shows that there is a strong correlation between cognitive style and relational understanding. In addition, the correlation is positive, so the better the GEFT test score, the better the relational understanding. Higher test scores indicate that students tend to have FI cognitive style.

Correlation between Relational Understanding (X₁) and Cognitive Style (X₂) with Problem-Solving Ability (Y)

To determine the correlation between relational understanding and cognitive style with problem-solving ability, a correlation analysis was performed using formula (2). The research showed that there is a significant correlation between relational understanding and cognitive style with problem-solving ability, evident from the calculation which resulted in $R_{yx_1x_2} = 0.965$. This shows that there is a strong correlation between relational understanding and cognitive style with problem-solving ability. Further, F test was performed to examine the significance of the multiple correlation coefficient, which resulted in $F_{count} = 128.24$ greater than $F_{table} = 3.52$ at a significance level of 5%. Therefore, it can be concluded that the multiple correlation coefficient is significant, meaning that the result of the correlation analysis can be applied to the population from which the samples were selected, i.e. high school students in Malang.

DISCUSSIONS

A strong correlation between relational understanding and cognitive style shows that these two factors can be considered in developing mathematical problem-solving ability. Students who have relational understanding tend to be better at problem-solving. Relational understanding helps students know more about what they learn and what they need in learning it (Anwar, 2016). The results also showed that a higher cognitive style score indicates a better problem-solving ability. Students with FI cognitive style tend to have a high level of independence in viewing stimulus without being dependent on teachers. On the other hand, students with FD cognitive style tend to be highly dependent on the educational resources from teachers (Lusiana, 2017). However, this does not necessarily mean that students with FI cognitive style are better than those with FD. If students with cognitive style are considered to have been more independent in solving problems, teachers are expected to be able to design and provide treatment needed by students with FD cognitive style. Finally, students with both FI and FD cognitive styles are expected to have the same opportunities to develop their problem solving ability.

CONCLUSIONS

Relational understanding and cognitive style have a significant correlation with problem-solving ability. The calculations resulted in $R_{yx_1x_2} = 0.965$, meaning that there is a strong correlation between relational understanding and cognitive style with problem-solving ability. The result of F test also showed that the multiple correlation coefficient is significant, indicating that the results of the correlation analysis can be applied to the population from which the samples were selected, i.e. high school students in Malang. These results indicate that in order to develop problem-solving ability, teachers shall consider students' relational understanding and cognitive style. For future researchers, it is suggested that they study how to design learning that could accommodate relational understanding or cognitive style as a part to improve students' problem-solving ability.

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